ՀՀ ԿՐԹՈՒԹՅԱՆ, ԳԻՏՈՒԹՅԱՆ, ՄՇԱԿՈՒՅԹԻ ԵՎ ՍՊՈՐՏԻ ՆԱԽԱՐԱՐՈՒԹՅՈՒՆ
ԵՐԵՎԱՆԻ ՊԵՏԱԿԱՆ ՀԱՄԱԼՍԱՐԱՆ

ՔՈԹԱՆՋՅԱՆ ՎԱՐԴԱԶԱՐ ԽՈՐԵՆԻ

ՎԱԿՈՒՈՒՄԱՅԻՆ ՔՎԱՆՏԱՅԻՆ ԵՐԵՎՈՒՅԹՆԵՐ ՈՉ-ԻՆԵՐՑԻԱԼ ՀԱՄԱԿԱՐԳԵՐՈՒՄ ԵՎ ԳՐԱՎԻՏԱՑԻՈՆ ԴԱՇՏԵՐՈՒՄ

Ա.04.02 - «Տեսական ֆիզիկա» մասնագիտությամբ
ֆիզիկամաթեմատիկական գիտությունների թեկնածուի
գիտական աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ - 2024

THE MINISTRY OF EDUCATION, SCIENCE, CULTURE AND SPORT OF RA
YEREVAN STATE UNIVERSITY

KOTANJYAN VARDAZAR

VACUUM QUANTUM EFFECTS IN NON-INERTIAL FRAMES AND GRAVITATIONAL FIELDS

Thesis for the degree of Candidate of Physical and Mathematical Sciences
Speciality 01.04.02 - “Theoretical Physics”

ABSTRACT

YEREVAN - 2024

Ատենախոսության թեման հաստատվել է Երևանի պետական համալսարանում

Գիտական ղեկավար՝ ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Ա. Ա. Սահարյան

Պաշտոնական ընդդիմախոսներ՝ ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Գ. Ս. Հաջյան
ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Ս. Դ. Օդինցով

Առաջատար կազմակերպություն՝ ՀՀ ԳԱԱ Վ. Համբարձումյանի անվան Բյուրականի աստղադիտարան

Ատենախոսության պաշտպանությունը կայանալու է 2024թ. մարտի 9-ին ժամը 12:00-ին Երևանի պետական համալսարանում գործող Ֆիզիկայի 049 Մասնագիտական խորհրդի նիստում:

Հասցե՝ 0025 Երևան, Ալեք Մանուկյան փ. 1, ԵՊՀ

Ատենախոսությանը կարելի է ծանոթանալ ԵՊՀ գրադարանում:

Սեղմագիրն առաքված է 2024թ. փետրվարի 7-ին:

Մասնագիտական խորհրդի գիտական քարտուղար՝ ֆիզ.-մաթ. գիտ. թեկնածու Վ. Պ. Քալանթարյան

The thesis theme is approved at the Yerevan State University.

Scientific supervisor: Doctor of Phys. Math. Sciences, Prof. A. A. Saharian

Official opponents: Doctor of Phys. Math. Sciences, Prof. G. S. Hajyan
Doctor of Phys. Math. Sciences, Prof. S. D. Odintsov

Leading organization: V. Ambartsumian Byurakan Astrophysical Observatory of NAS RA

The defence of the thesis will take place at 12:00 on March 9, 2024, during the session of the Specialized Council 049 of Physics at the Yerevan State University.

Address: 1 Alex Manoogian Street, 0025 Yerevan, Armenia.

The thesis is available in the Yerevan State University library.

The abstract was distributed on 7 February, 2024.

Scientific secretary of the Specialized Council Candidate of Phys. Math. Sciences V. P. Kalantaryan

GENERAL DESCRIPTION OF THE WORK

Relevance of the topic. Among the important lessons from quantum field theory on curved spacetimes is the dependence of the vacuum and particle notions on the observer. The natural mode functions used in the canonical quantization of fields may differ for different observers, giving rise to different particle and vacuum states. If the Bogoliubov transformations mix the annihilation and creation operators corresponding to two sets of mode functions, the Bogoliubov β-coefficient is different from zero, and the vacuum states based on those sets are not equivalent: the vacuum state corresponding to one set of modes contains particles of the other set of modes. The classical examples of two inequivalent vacuum states in flat spacetime are the Minkowski and Fulling-Rindler (FR) vacua. They are vacuum states for inertial and uniformly accelerating observers, respectively. The interest in the quantization of fields in Rindler coordinates, which are the natural coordinates for uniformly accelerating observers, is motivated by several reasons. Firstly, it comes from the principal questions of quantization of fields in geometries having horizons. The latter can be either observer-dependent (like Rindler or de Sitter (dS) horizons) or determined by the matter distribution (examples are the black hole horizons). The Rindler geometry is simple enough to allow exact solutions to different problems of quantum field theory to be found. This may shed light on the problems in more complicated geometries, where the exact solutions are not available or are complicated. Next, the Rindler metric approximates the black hole geometry in the near horizon limit, and the roots of several quantum field theoretical phenomena around black holes can be found in the Rindler physics. An example is the relation between the Unruh effect and Hawking radiation. Being a background with horizons, the Rindler geometry is an interesting arena to investigate the phenomena of quantum entanglement.

Among the exciting effects of quantum field theory in curved backgrounds are the polarization of the vacuum and particle creation by gravitational fields. In particular, these effects play an essential role in the early Universe and near the black holes. An essential feature of vacuum polarization in the presence of gravitational fields is the possibility of breaking the energy conditions in Hawking-Penrose singularity theorems. This provides an opportunity to solve the singularity problem in General Relativity. The exact results for the physical characteristics describing the vacuum polarization are obtained for highly symmetric gravitational fields. In particular, the dS and anti-de Sitter (AdS) spacetimes have attracted a great deal of attention. Having the same number of symmetries as the Minkowski spacetime, these geometries are maximally symmetric. They are vacuum solutions of the Einstein equations with positive (dS) and negative (AdS) cosmological constants as the only source of the gravitational field. The results for the influence of the gravitational fields on quantum matter, obtained for dS and AdS bulks, may shed light on the effects of gravity in more complicated background geometries. In addition to this, the popularity of the dS and AdS spacetimes in quantum field theory is motivated by their important role in cosmology and in high-energy models with extra dimensions.

In accordance with the inflationary scenario, at the beginning of the evolution, the Universe went through a phase of accelerating expansion, which is approximated by the dS spacetime. Several models have been proposed for the inflationary phase, the predictions of which can be tested by using the recent observations of temperature anisotropies of the cosmic microwave background radiation. On the other hand, the observational data on those anisotropies, high redshift supernovae, and galaxy clusters indicate that the recent expansion of the Universe is dominated by a source of the cosmological constant type. The relative contribution of the latter to the total energy density will increase during the expansion, and the corresponding geometry asymptotes dS spacetime in the future. Hence, the dS geometry appears as the past and future attractors for the expansion of the Universe. The dynamics of quantum fields in AdS spacetime have long been an active field of research. The early interest was related to the principal questions of the quantization procedure on curved backgrounds. The lack of global hyperbolicity and the presence of both regular and irregular modes were among the new features having no analogues in quantum field theory on the Minkowski bulk. The natural appearance of AdS spacetime as a ground state in supergravity and Kaluza-Klein theories and also as the near-horizon geometry of the extremal black holes and domain walls has further increased the interest in quantum field theories on AdS bulk. The AdS spacetime has played an important role in two exciting developments of theoretical physics during the last decade: AdS/conformal field theory (CFT) correspondence and braneworld scenarios with large extra dimensions. The AdS/CFT correspondence establishes a duality between two different theories: supergravity or string theory on asymptotically AdS bulk from one side and conformal field theory on AdS boundary from another. It provides a vital possibility to investigate strong coupling non-perturbative effects in one theory by mapping them to the weak coupling region of dual theory. It has been applied in different physical settings, including a variety of condensed matter systems. The braneworld paradigm naturally arises in the context of supergravity and string theories and presents an alternative to Kaluza-Klein compactification of extra dimensions. The models formulated on AdS bulk provide a geometrical solution for the hierarchy problem between the electroweak and gravitational energy scales, and also new perspectives and different interpretations for various problems in particle physics and cosmology.

The aim of the thesis is to investigate the influence of the background gravitational field, the spatial topology and boundaries on the properties of the vacuum state for quantum scalar, fermionic and electromagnetic fields. In accordance with the equivalence principle, the features of the local influence of the gravitational field can be studied considering the physical processes in non-inertial reference frames. The thesis includes the investigations of the properties of the vacuum in a uniformly accelerating reference frame (FR vacuum). The following problems are considered.

- Investigation of the vacuum expectation values (VEVs) of the field squared and current density for a charged scalar field in the Rindler spacetime with a part of spatial dimensions compactified to a torus.

- Investigation of the properties of the fermionic FR vacuum in a general number of spatial dimensions. As local characteristics of the vacuum state, the fermion condensate (FC) and the VEV of the energy-momentum tensor are considered, and their dependence on the spatial dimension is studied.

- Properties of the electromagnetic vacuum around a topological defect in dS spacetime that generalizes the geometry of a cosmic string for a general number of spatial dimensions.

- Investigation of the quantum vacuum effects for a massive scalar field with general curvature coupling parameter induced by two parallel branes orthogonal to the AdS boundary. The two-point function, the VEVs of the field squared and energy-momentum tensor, and the Casimir forces are studied.

Scientific novelty. The expectation values of the field squared and current density for a charged scalar field in the FR vacuum are examined in Rindler spacetime with a compact subspace. Scalar mode functions are found for general quasiperiodicity phases and the expression of the Hadamard function is derived, where the corresponding function for the Minkowski vacuum is explicitly extracted. The VEVs are periodic functions of the magnetic flux enclosed by compact dimensions, with the period equal to the flux quantum. The current density tends to zero on the Rindler horizon. The difference in the current densities for the FR and Minkowski vacua is exponentially small for small accelerations and lengths of compact dimensions. The local properties of the FR vacuum are studied for a massive Dirac field in a general number of spatial dimensions. Corresponding fermionic normal modes are found, and the renormalized FC and the VEV of the energy-momentum tensor are explored. The FC vanishes for massless fields and is negative for massive fields. The properties of the vacuum energy-momentum tensor near the Rindler horizon are examined, showing weak dependence on the mass for large accelerations and an exponential decay for small accelerations. The effects of background geometry and topology on the VEV of the energy-momentum tensor for the electromagnetic field in locally dS spacetime are studied. The non-trivial topology is induced by a defect, which is a generalization of a cosmic string. The topological contribution in the vacuum energy-momentum tensor is explicitly extracted. Outside the defect core the renormalization is required only for the pure dS part. The asymptotic behaviour of the topological contribution at small and large distances from the defect core is studied. The influence of two parallel branes, orthogonal to the AdS boundary, on the scalar vacuum in AdS spacetime is investigated. General Robin boundary conditions are imposed on separate branes, and the positive frequency Wightman function is evaluated. The vacuum energy density can be either negative or positive depending on the boundary conditions and the distance from the branes. The asymptotics for points near and far from the branes are discussed in detail. The normal and shear components of the Casimir forces, acting on the branes, are investigated. Their signs depend on the Robin coefficients, the distance from the branes and the distance from the AdS boundary.

Practical importance. The mode functions for a scalar field in Rindler spacetime with a part of spatial dimensions compactified to a torus can be used for the investigation of the effects of non-trivial topology on the VEV of the energy-momentum tensor. The corresponding Wightman function can be employed in studying the response of Unruh-de Witt-type particle detectors in a given state of motion. The fermionic Rindler modes presented in the thesis are easily generalized for spacetimes with toroidally compact dimensions and general quasiperiodic conditions along them. With this generalization, those modes can be used to investigate the fermionic current density. In the particular case of two-dimensional space, the corresponding results can be applied to carbon nanotubes within the framework of the Dirac model for a long-wavelength description of the corresponding electronic subsystem. The results for the vacuum energy-momentum tensor around a cosmic string can be used to investigate the back reaction of quantum effects on the background geometry. Those results are also important considering the evolution of vacuum fluctuations in the post-inflationary era of the Universe's expansion. The results obtained for the geometry of branes in AdS spacetime should be taken into account when considering the stability of corresponding braneworlds. Within the framework of the AdS/CFT correspondence, those results can also be used for the investigation of the boundary-induced effects in conformal field theory living on the boundary of AdS spacetime.

The basic results to be defended are as follows:

1. The magnetic fluxes enclosed by compact spatial dimensions generate currents in the FR vacuum state for a charged scalar field in locally Rindler spacetime. The charge density and the components of the current along uncompact dimensions vanish. The components of the current in the compact subspace are periodic functions of the magnetic flux, with the period equal to the flux quantum. They tend to zero on the Rindler horizon. The expectation value of the field squared in the FR vacuum is an even periodic function of the magnetic flux. Depending on the value of the proper acceleration, it can be either positive or negative.

2. For a massive Dirac field, the fermion condensate is negative in the FR vacuum state for the general case of the spatial dimension and tends to zero in the massless limit. The corresponding VEV of the energy-momentum tensor is diagonal, and the vacuum stresses are isotropic. Compared to the Minkowski vacuum, the energy density and the effective pressures in the FR vacuum are negative. For a massless field, the corresponding equation of state is of the radiation type and the spectral distribution is thermal with the Unruh temperature. The thermal

distribution is of the Fermi-Dirac and Bose-Einstein types in odd and even numbers of spatial dimensions, respectively.

3. For the Bunch-Davies vacuum state in locally dS spacetime and in the presence of a generalized cosmic string type defect, the expectation value of the energy-momentum tensor for the electromagnetic field is not diagonal in spatial dimensions $D > 3$. The off-diagonal component corresponds to the vacuum energy flux along the radial direction with respect to the defect core. It is directed towards the cosmic string. The contributions in the vacuum stresses induced by the cosmic string are anisotropic, and for $D > 3$ the stresses along the directions parallel to the string core differ from the energy density. Depending on the planar angle deficit and the distance from the cosmic string, the vacuum energy density and pressures can be positive or negative. The influence of the gravitational field on the diagonal components of the energy-momentum tensor is weak for points near the cosmic string and is essential at proper distances from the cosmic string larger than the dS curvature radius. At large distances, the topological contributions in the diagonal components tend to zero like the inverse fourth power of the proper distance and the energy flux behaves as the inverse-fifth power for all values of the spatial dimension. The exception is the energy density in the special case $D = 4$.

4. The vacuum energy-momentum tensor for a massive quantum scalar field in the geometry of two parallel branes perpendicular to the AdS boundary has a non-zero off-diagonal stress. Depending on the coefficients in the Robin boundary conditions on the branes and the distance from the branes, the vacuum energy density can be either positive or negative. The off-diagonal component of the vacuum stress gives rise to the component of the Casimir force parallel to the branes (shear force). If the boundary conditions on the separate branes are different, the corresponding normal Casimir forces differ, and they can be either repulsive or attractive. Depending on the coefficients in the boundary conditions, the shear force is directed toward or from the AdS boundary. At large proper separations between the branes, compared to the AdS curvature radius, both of the components of the Casimir forces exhibit a power-law decay.

Approbation of the work. The results of the thesis were reported at the conferences "Modern Physics of Compact Stars and Relativistic Gravity" (Yerevan, 2021, 2023) and have been discussed at the seminars of the Chair of Theoretical Physics of Yerevan State University and of the INFN National Laboratory of Frascati (Frascati, Italy).

Publications. Five papers were published on the topic of the thesis.

Structure of the thesis. The thesis consists of an Introduction, three Chapters, a Conclusion, six Appendices and a bibliography. It contains 172 pages, including 31 figures.

CONTENT OF THE THESIS

In **Introduction**, the scientific literature related to the topic of the thesis is reviewed, the relevance of the topic is argued, the aim of the work, the scientific novelty and the practical value are presented, and the primary results are described.

In **Chapter 1**, the VEV of the field squared and the vacuum currents are investigated for a charged scalar field in Rindler spacetime with a toroidally compact subspace. In the Rindler coordinates, the geometry is described by the $(D+1)$-dimensional line element (units with $c = \hbar = 1$ are used) $ds_{\mathrm{R}}^2 = \rho^2 d\tau^2 - d\rho^2 - d\mathbf{x}^2$, where $d\mathbf{x}^2 = \sum_{i=2}^{D} (dx^i)^2$ and $0 < \rho < \infty$. The worldline for given $(\rho, x^2, \dots, x^D)$ corresponds to an observer with constant proper acceleration $1/\rho$. The p-dimensional subspace covered by Cartesian coordinates $\mathbf{x}_p = (x^2, ..., x^{p+1})$ has trivial topology, R^p, with the range of variations $-\infty < x^l < \infty$ for $l = 2, .., p+1$. The subspace corresponding to the coordinates $\mathbf{x}_q = (x^{p+2}, ..., x^D)$ is compactified to a q-dimensional torus $(S^1)^q$, $q = D - p - 1$. The length of the compact dimension x^l will be denoted by L_l and one has $0 \leqslant x^l \leqslant L_l$ for $l = p+2, ..., D$. Assuming the presence of an external classical gauge field with the vector potential A_μ, the field equation for a quantum charged scalar field $\varphi(x)$ reads $(g^{\mu\nu} D_\mu D_\nu + m^2)\, \varphi = 0$, with spacetime point $x = (\tau, \rho, \mathbf{x})$ and gauge extended covariant derivative $D_\mu = \nabla_\mu + ieA_\mu$. Generic quasiperiodic conditions $\varphi(t, \xi, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\alpha_l} \varphi(t, \xi, \mathbf{x}_p, \mathbf{x}_q)$ are imposed along compact dimensions, where $l = p+2, \ldots, D$, and $\mathbf{e}_l$ is the unit vector along the dimension x^l. By using the complete set of the mode functions, obeying those conditions, a closed expression of the Hadamard function $G(x, x')$ for the FR vacuum is derived for a gauge field with constant components A_l along compact dimensions.

Given the Hadamard function, the VEV of the field squared is evaluated by using the relation $\langle \varphi(x) \varphi^\dagger(x) \rangle = \lim_{x' \to x} G(x, x')/2$. The corresponding expression reads

$$\langle \varphi\varphi^\dagger \rangle = \langle \varphi\varphi^\dagger \rangle_{\mathrm{M}} - \frac{m^{D-1}}{(2\pi)^{\frac{D+1}{2}}} \sum_{\mathbf{n}_q} \cos(\mathbf{n}_q \cdot \tilde{\boldsymbol{\alpha}}) \int_0^\infty du \frac{f_{\frac{D-1}{2}}(m\sqrt{4\rho^2 \cosh^2 u + \sum_{i=p+2}^{D} L_i^2 n_i^2})}{u^2 + \pi^2/4}, \quad (1)$$

where $\mathbf{n}_q = (n_{p+2}, n_{p+3}, \ldots, n_D)$, $\tilde{\boldsymbol{\alpha}} = (\tilde{\alpha}_{p+2}, \ldots, \tilde{\alpha}_D)$, $\mathbf{n}_q \cdot \tilde{\boldsymbol{\alpha}} = \sum_{l=p+2} n_l \tilde{\alpha}_l$, and $\tilde{\alpha}_l = \alpha_l + eA_l L_l$. In (1), $\sum_{\mathbf{n}_q} = \sum_{n_{p+2}=-\infty}^{+\infty} \cdots \sum_{n_D=-\infty}^{+\infty}$ and the function $f_\nu(x)$ is defined by $f_\nu(x) = x^{-\nu} K_\nu(x)$, where $K_\nu(x)$ is the modified Bessel function. The Minkowskian VEV $\langle \varphi\varphi^\dagger \rangle_{\mathrm{M}}$ is found by using the finite temperature Hadamard function from [1]. The renormalization is required only for that part. Note that the term $eA_l L_l$ in the expression for $\tilde{\alpha}_l$ can be written as $eA_l L_l = -2\pi \Phi_l/\Phi_0$, where $\Phi_0 = 2\pi/e$ is the flux quantum and Φ_l is the magnetic flux enclosed by the lth compact dimension. The Rinlder horizon corresponds to the limit $\rho \to 0$ (large accelerations). Near the horizon the last term in (1) dominates and the VEV $\langle \varphi\varphi^\dagger \rangle$ behaves as $1/\rho^{D-1}$. For small accelerations, corresponding to large values of ρ, the difference $\langle \varphi\varphi^\dagger \rangle - \langle \varphi\varphi^\dagger \rangle_{\mathrm{M}}$ is suppressed by the exponential factor $e^{-2\rho\omega_0}$, where $\omega_0^2 = \sum_{i=p+2}^{D} \tilde{\alpha}_i^2/L_i^2 + m^2$ and it is assumed that $|\tilde{\alpha}_i| \leqslant \pi$.

The VEV of the current density for a charged scalar field is obtained from the formula $\langle j_\mu \rangle = ie \lim_{x' \to x} (\partial_\mu - \partial'_\mu) G(x, x')/2$. The charge density and the components along uncompact dimensions vanish: $\langle j_\mu \rangle = 0$, $\mu = 0, 1, \ldots, p+1$. The contravariant component along the lth compact dimension is given by the expression

$$\langle j^l \rangle = \langle j^l \rangle_{\mathrm{M}} - \frac{2em^{D+1} L_l}{(2\pi)^{\frac{D+1}{2}}} \sum_{\mathbf{n}_q} n_l \sin(\mathbf{n}_q \cdot \tilde{\boldsymbol{\alpha}}) \int_0^\infty du \frac{f_{\frac{D+1}{2}}(m\sqrt{4\rho^2 \cosh^2 u + \sum_{i=p+2}^{D} L_i^2 n_i^2})}{u^2 + \pi^2/4}, \quad (2)$$

with $l = p+2, \ldots, D$, and $\langle j^l \rangle_{\mathrm{M}}$ being the current density for the Minkowski vacuum. The current density (2) is an odd periodic functions of $\tilde{\alpha}_l$ with the period 2π and even periodic functions of $\tilde{\alpha}_i$, $i \neq l$,with the same period. This corresponds to the periodicity in the magnetic flux Φ_l. The current density vanishes on the Rindler horizon, $\lim_{\rho \to 0} \langle j^l \rangle = 0$. Similar to the case of the field squared, the difference of the current densities for FR and Minkowski vacua are exponentially small for small acceleration, $\langle j^l \rangle - \langle j^l \rangle_{\mathrm{M}} \propto e^{-2\rho\omega_0}$, $\rho\omega_0 \gg 1$. In figure 1 the graphs for the current density are plotted for the model $D = 4$ with a single compact dimension of the length L and $\tilde{\alpha}_D/2\pi = 0.2$. The left panel presents the dependence of the current density on $m\rho$ for different values of mL (the numbers near the curves). In the limit $m\rho \to \infty$ the current density tends to $\langle j^D \rangle_{\mathrm{M}}$ and it vanishes on the Rindler horizon $\rho = 0$. On the right panel, the difference in the current densities for the FR and Minkowski vacua (in units of em^D) is plotted versus the length of the compact dimension. The numbers near the curves are the values for $m\rho$.

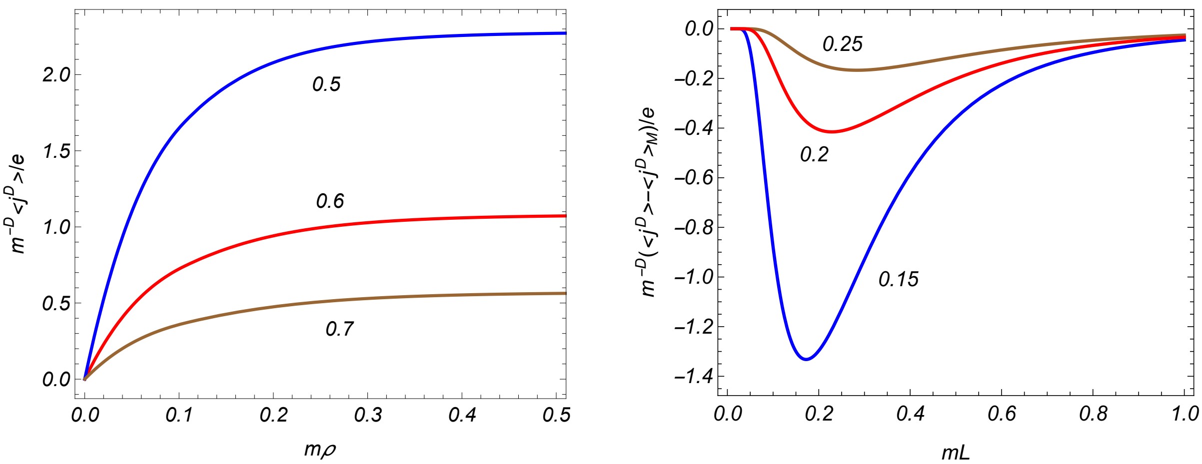

Figure 1: The current density as a function of the inverse acceleration and the length of compact dimension.

As an application, the vacuum currents near the horizon of cylindrical black holes are investigated. The corresponding exterior geometry is approximated by the Rindler-like metric considered above with the lengths of the compact dimensions $L_l = 2\pi r_H$, where r_H is the radius of the event horizon. At large distances from the horizon, the geometry of cylindrical black holes is approximated by a locally AdS spacetime with a toroidally compact subspace. The lengths of the corresponding compact dimensions are expressed in terms of the AdS curvature scale a as $L_l = 2\pi a$. The vacuum currents in the latter geometry have been investigated in [2].

In the second part of **Chapter 1**, the local properties of the FR vacuum for a massive Dirac field $\psi(x)$ are investigated in a general number of spatial dimensions. As characteristics, the FC and the VEV of the energy-momentum tensor are considered. The mode summation technique is employed in combination with the point-splitting regularization procedure to evaluate those quantities. The line element is the same as that for a scalar field, but now for all spatial dimensions $-\infty < x^l < \infty$, $l = 2, 3, \ldots, D$. The dynamics of the field is described by the Dirac equation $(i\gamma^\mu \nabla_\mu - m)\,\psi = 0$, where $\nabla_\mu = \partial_\mu + \Gamma_\mu$ and Γ_μ is the spin connection. A fermionic field, realizing the irreducible representation of the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, is considered with $N \times N$ Dirac matrices, where $N = 2^{[(D+1)/2]}$. As the first step, the results of [3] are generalized for the complete set of fermionic normal modes in Rindler spacetime in the case of a massive field in $(D+1)$-dimensional spacetime. By using those modes, the following formula is obtained for the FC in the FR vacuum:

$$\langle \bar{\psi}\psi \rangle_{\rm FR} = -\frac{2Nm^D}{(2\pi)^{\frac{D+3}{2}}} \int_0^\infty du\, \frac{u \sinh u}{u^2 + \pi^2/4} f_{\frac{D-1}{2}}\left(2m\rho \cosh u\right), \tag{3}$$

where $\bar{\psi} = \psi^\dagger \gamma^0$. From (3) it follows that the FC vanishes for a massless field. In the case of a massive field, the condensate is negative and monotonically decreases with increasing proper acceleration $1/\rho$. It is exponentially suppressed by the factor $e^{-2m\rho}$ for large values of ρ and behaves like $1/\rho^{D-1}$ for small ρ (near the Rindler horizon).

The renormalized VEV of the energy-momentum tensor in the FR vacuum is diagonal. The corresponding energy density and the vacuum stresses are expressed as (no summation over $l = 1, \ldots, D$)

$$\begin{aligned} \langle T_0^0 \rangle_{\rm FR} &= -\frac{2Nm^{D+1}}{(2\pi)^{\frac{D+3}{2}}} \int_0^\infty du\, \frac{u \sinh u}{u^2 + \pi^2/4} \left[f_{\frac{D-1}{2}}(2m\rho \cosh u) + D f_{\frac{D+1}{2}}(2m\rho \cosh u) \right], \\ \langle T_l^l \rangle_{\rm FR} &= \frac{2Nm^{D+1}}{(2\pi)^{\frac{D+3}{2}}} \int_0^\infty du\, \frac{u \sinh u}{u^2 + \pi^2/4} f_{\frac{D+1}{2}}\left(2m\rho \cosh u\right). \end{aligned} \tag{4}$$

The vacuum effective pressures are determined by $-\langle T_l^l \rangle_{\rm FR}$ and they are isotropic. Note that the latter property generally does not take place for a scalar field. From (4) it follows that the energy density and the effective pressures in the FR vacuum are negative. For a massless field, the general expressions (4) are simplified to

$$\langle T_\mu^\nu \rangle_{\rm FR} = \frac{2^{-D} N \rho^{-D-1}}{\pi^{\frac{D}{2}} \Gamma\left(\frac{D}{2}\right)} \int_0^\infty d\omega \, \frac{\omega^D B_D(\omega)}{e^{2\pi\omega} - (-1)^D} \mathrm{diag}\left(-1, \frac{1}{D}, \ldots, \frac{1}{D}\right), \tag{5}$$

where $B_0 = B_1 = 1$ and

$$B_D(\omega) = \prod_{l=1}^{l_D} \left[1 + \left(\frac{l - \{D/2\}}{\omega}\right)^2\right], \tag{6}$$

for $D \geq 2$. Here, $\{D/2\}$ is the fractional part of $D/2$ and $l_D = D/2 - 1 + \{D/2\}$. Note that the energy ε_ρ measured by an observer with proper acceleration $1/\rho$ is expressed as $\varepsilon_\rho = \omega/\rho$. The respective factor $\left(e^{2\pi\rho\varepsilon_\rho} + 1\right)^{-1}$ is interpreted as an indication of the thermal nature of inertial vacuum with respect to a uniformly accelerating observer. The corresponding temperature (Unruh temperature) is given by $T = T_U = 1/(2\pi\rho)$. An interesting point to be mentioned is that in an even number of spatial dimensions, the thermal factor for the Dirac field is of bosonic type, $(e^{2\pi\rho\varepsilon_\rho} - 1)^{-1}$. Similar features in the response of a uniformly accelerating Unruh-DeWitt detector interacting with the Dirac field prepared in the Minkowski vacuum have been observed previously in the literature.

For a massive fermionic field and small accelerations corresponding to $m\rho \gg 1$, the energy density and the stresses are exponentially suppressed: $\langle T_0^0 \rangle_{\rm FR} \propto (m\rho)^{-\frac{D+3}{2}} e^{-2m\rho}$ and $\langle T_l^l \rangle_{\rm FR} \approx -2m\rho \langle T_0^0 \rangle_{\rm FR}$, $l = 1, \ldots, D$. An interesting feature is seen in the asymptotic estimate is that the absolute value of the energy density is much smaller than the absolute value of the pressure, $|\langle T_0^0 \rangle_{\rm FR}| \ll |\langle T_1^1 \rangle_{\rm FR}|$. For classical sources $T_0^0 \geq |T_1^1|$ and in the non-relativistic limit $|T_1^1| \ll T_0^0$. In the opposite limit $m\rho \ll 1$, the leading term in the expansion over $m\rho$ coincides with the VEV for a massless field. In figure 2 the FC and the energy density in the FR vacuum are plotted as functions of dimensionless combination $m\rho$ for different values of the spatial dimension (numbers near the curves). The qualitative behaviour for the vacuum effective pressure $-\langle T_1^1 \rangle_{\rm FR}$ is similar to that for the energy density.

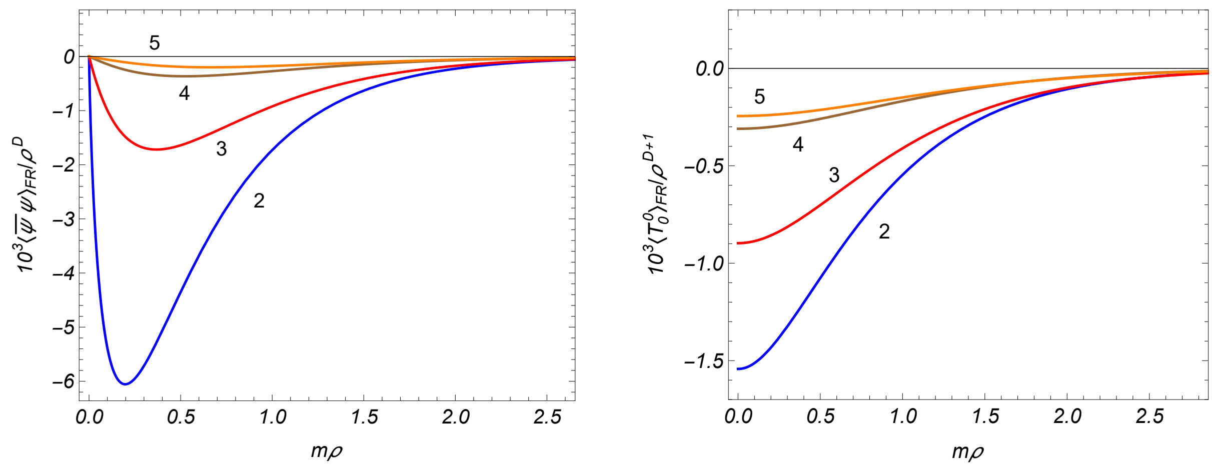

Figure 2: The FC and the energy density versus the inverse proper acceleration.

Having the VEVs for a massless Dirac field in the FR vacuum, the corresponding VEVs are generated in the problems where the background geometry is conformally related to the Rindler spacetime.

As such geometries, a static spacetime with constant negative curvature spatial sections, the Milne universe, dS spacetime foliated by negative curvature spatial sections and dS spacetime described in static coordinates are considered.

In **Chapter 2**, the combined effects of the background geometry and topology on the VEV of the energy-momentum tensor for the electromagnetic field in $(D+1)$-dimensional locally dS spacetime are studied. The non-trivial topology is generated due to a topological defect generalising a cosmic string in 4-dimensional spacetime. The corresponding geometry is described by the line element $ds_{\text{dS}}^2 = dt^2 - e^{2t/\alpha}[dr^2 + r^2 d\phi^2 + (d\mathbf{z})^2]$, where $\mathbf{z} = \left(z^3, ..., z^D\right)$, $t, z^i \in (-\infty, +\infty)$, $0 \leqslant r < \infty$, and $0 \leqslant \phi \leqslant \phi_0$. In the special case $\phi_0 = 2\pi$ the geometry corresponds to the $(D+1)$-dimensional dS spacetime sourced by the cosmological constant $\Lambda = D(D-1)/(2\alpha^2)$. For $\phi_0 < 2\pi$ and at the points $r > 0$ the local characteristics coincide with those for dS spacetime. The defect core, given by $r = 0$, presents a $(D-2)$-dimensional spatial hypersurface. In terms of the conformal time coordinate $\tau = -\alpha e^{-t/\alpha}$, the metric tensor is given as $g_{il} = (\alpha/\tau)^2 \operatorname{diag}(1, -1, -r^2, -1, \dots, -1)$. The complete set of the normalized mode functions is found for the vector potential $A_\mu(x)$ of the electromagnetic field. By employing those modes, the VEV of the energy-momentum tensor for the electromagnetic field, $\langle T_i^l \rangle$, is investigated, assuming that the field is prepared in the state which is the analogue of the Bunch-Davies vacuum state in dS spacetime. The contribution in the energy-momentum tensor induced by the presence of the defect (the topological part) is given by $\langle T_i^l \rangle_{\text{t}} = \langle T_i^l \rangle - \langle T_i^l \rangle^{(\text{dS})}$, where $\langle T_i^l \rangle^{(\text{dS})}$ is the VEV of the energy-momentum tensor in dS spacetime in the absence of the cosmic string. From the maximal symmetry of the Bunch-Davies state, it follows that $\langle T_i^l \rangle^{(\text{dS})} = \text{const} \cdot \delta_i^l$.

For the topological contribution in the VEVs of the nonzero components of the energy-momentum tensor, the following expressions are obtained (no summation over i):

$$\langle T_i^i \rangle_{\text{t}} = \frac{2\alpha^{-D-1}}{(2\pi)^{D/2+1}} \left[\sum_{j=1}^{[q/2]}{}' t^{(i)}\left(r/\eta, s_j\right) - \frac{q}{\pi}\sin(q\pi) \int_0^\infty dz \frac{t^{(i)}\left(r/\eta, \cosh z\right)}{\cosh(2qz) - \cos(q\pi)} \right], \tag{7}$$

$$\langle T_0^1 \rangle_{\text{t}} = \frac{8(D-3)r}{(2\pi)^{D/2+1}\alpha^{D+1}\eta} \left[\sum_{j=1}^{[q/2]} t^{(01)}(r/\eta, s_j) - \frac{q}{\pi}\sin(q\pi) \int_0^\infty dy \frac{t^{(01)}(r/\eta, \cosh y)}{\cosh(2qy) - \cos(q\pi)} \right], \tag{8}$$

where $\eta = |\tau|$, $q = 2\pi/\phi_0$, $s_j = \sin(j\pi/q)$ and the prime on the summation sign means that the term $j = q/2$ (for even values of q) should be taken with additional coefficient $1/2$. In (7), the notations

$$t^{(i)}(x, y) = \int_0^\infty du u^{\frac{D}{2}} e^{u - 2ux^2y^2} \sum_{l=1,2} K_{\frac{D}{2}-l}(u) f_l^{(i)}(x, y, u), \tag{9}$$

$$t^{(01)}(x, y) = y^2 \int_0^\infty du\, u^{\frac{D}{2}} \left(1 - uy^2x^2\right) K_{\frac{D}{2}-1}(u) e^{u - 2x^2y^2u} \tag{10}$$

are introduced with the functions

$$\begin{aligned} f_l^{(i)}(x, y, u) &= \left[4b_l^{(i)} ux^2y^2 - 2(b_l^{(i)} + d_l^{(i)})ux^2 - ((D-2)a_l^{(i)} + 2b_l^{(i)})\right] y^2 + e_l^{(i)},\ i \neq 1, 2, \\ f_l^{(i)}(x, y, u) &= 2(b_l^{(i)} - d_l^{(i)})ux^2y^2 + e_l^{(i)},\ i = 1, 2. \end{aligned} \tag{11}$$

The numerical coefficients $a_l^{(i)}$, $b_l^{(i)}$, $c_l^{(i)}$, $d_l^{(i)}$, and $e_l^{(i)}$ are completely determined by the spatial dimension D. The off-diagonal component $\langle T_0^1 \rangle_{\text{t}}$ corresponds to the energy flux along the radial direction. For this component one has $\langle T_0^1 \rangle_{\text{t}} < 0$ which means that the flux is directed towards the cosmic string. The topological part of the VEV depends on η and r in the form of the ratio r/η. This property is a consequence of the maximal symmetry of dS spacetime. Considering that $\alpha r/\eta$ is the proper distance from the string, it is seen that r/η is the proper distance, measured in units of the dS curvature scale α.

For odd values of D, the topological contribution is expressed in terms of elementary functions. In particular, for $D=3,5$ one gets (no summation over i)

$$\langle T_i^i\rangle_{\rm t} = -\frac{A^{(i)}c_4(q)}{8\pi^2(\alpha r/\eta)^4},\ \langle T_0^1\rangle_{\rm t}=0,\ A^{(0)}=A^{(1)}=A^{(3)}=1,\ A^{(2)}=-3,\ D=3, \tag{12}$$

$$\langle T_i^i\rangle_{\rm t} = \frac{\left[B^{(i)}+(r/\eta)^2C^{(i)}\right]c_4(q)+D^{(i)}c_6(q)}{16\pi^3(\alpha r/\eta)^6},\ \langle T_0^1\rangle_{\rm t}=-\frac{c_4(q)}{8\pi^3\alpha^6\,(r/\eta)^5},\ D=5, \tag{13}$$

where $B^{(0)}=B^{(i)}=2$, $B^{(1)}=B^{(2)}=0$, $C^{(0)}=-C^{(1)}=1$, $C^{(2)}=5$, $D^{(0)}=D^{(1)}=-2$, $D^{(2)}=10$, and

$$c_4(q)=\frac{q^2-1}{90}(q^2+11),\ c_6(q)=\frac{q^2-1}{1890}(2q^4+23q^2+191). \tag{14}$$

At small proper distances compared to the curvature radius of the background spacetime, one has $r/\eta\ll 1$, and the leading terms in the asymptotic expansions of the diagonal components near the string coincide with the corresponding expressions for the Minkowski bulk with the distance from the string replaced by the proper distance in the dS bulk. In that region, $\langle T_i^i\rangle_{\rm t}\propto 1/(\alpha r/\eta)^{D+1}$ with the relations $\langle T_l^l\rangle_{\rm t}\approx\langle T_0^0\rangle_{\rm t}$, $l=3,\ldots,D$, and $\langle T_2^2\rangle_{\rm t}\approx -D\langle T_1^1\rangle_{\rm t}$. The energy density near the cosmic string is negative in spatial dimensions $D=3,4$. Near the cosmic string, the radial stress is negative, and the azimuthal stress is positive, $\langle T_1^1\rangle_{\rm t}<0$ and $\langle T_2^2\rangle_{\rm t}>0$. For the off-diagonal component one has $\langle T_0^1\rangle_{\rm t}\propto(\alpha r/\eta)^{-D}/\alpha$ for $r/\eta\ll 1$. At large distances from the cosmic string, one has $r/\eta\gg 1$ and the leading terms in the corresponding asymptotic expansions are given by (no summation over i)

$$\langle T_i^i\rangle_{\rm t}\approx\frac{\Gamma(D/2-1)t_0^{(i)}c_4(q)}{32\pi^{D/2+1}\alpha^{D+1}(r/\eta)^4},\ \langle T_0^1\rangle_{\rm t}\approx-\frac{(D-3)\Gamma(D/2-1)c_4(q)}{8\pi^{D/2+1}\alpha^{D+1}\,(r/\eta)^5}, \tag{15}$$

where $\Gamma(x)$ is the gamma function and the numerical coefficients $t_0^{(i)}$ are completely determined by the spatial dimension D. The exception is the energy density for the case $D=4$ with the asymptotic $\langle T_0^0\rangle_{\rm t}\approx -3c_6(q)\ln(r/\eta)/[8\pi^3\alpha^4(r/\eta)^6]$. At large distances from the cosmic string, the topological contribution to the energy density is negative for $D=3,4$ and positive for $D>4$. The stresses $\langle T_i^i\rangle_{\rm t}$, with $i=1,3,...,D$, are negative for $3\le D\le 6$ and positive for $D>6$. The stress $\langle T_2^2\rangle_{\rm t}$ is positive for $D\ge 3$ and the energy flux is negative. Notably, the topological contributions in the diagonal components decay at large distances as the inverse fourth power of the proper distance from the cosmic string in all spatial dimensions $D\ge 3$. The exception is the energy density in 4-dimensional space. This behaviour is in contrast to the geometry of a defect in the Minkowski bulk where the VEV decays like $1/r^{D+1}$.

Figure 3 displays the dependence of the diagonal components of the topological contributions in the VEV of the energy-momentum tensor, $\langle T_i^i\rangle_{\rm t}$ (in units of $1/\alpha^{D+1}$), on the ratio r/η (proper distance from the cosmic string in units of the dS curvature radius α). The numbers near the curves correspond to the value of the index i, and the left and right panels are plotted for $D=5$ and $D=6$, respectively. For For the planar angle deficit, we have taken the value corresponding to $q=1.5$. The dashed curves on both panels present the energy flux. For both cases, $D=5,6$, the energy density is positive. For $D=3,4$, the corresponding energy density is negative. In general, depending on the values of D and q, the energy density can be either positive or negative. For the values of the parameters corresponding

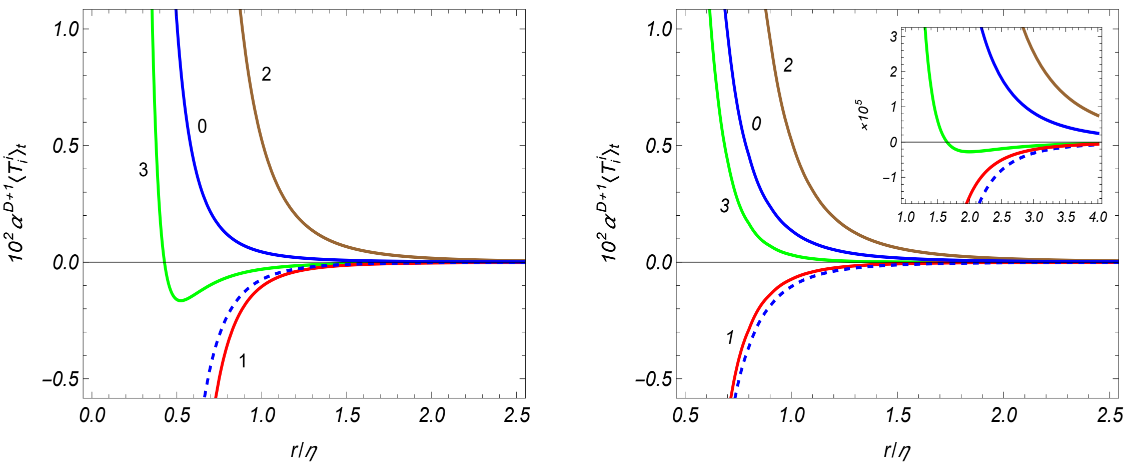

Figure 3: The radial dependence of the topological contributions in the vacuum energy-momentum tensor.

to figure 3 the radial and azimuthal stresses are monotonic functions of r/η, whereas the stresses $\langle T_i^i\rangle_{\rm t}$, $i = 1, 3, \ldots, D$, are positive near the cosmic string and negative at large distances.

Chapter 3 considers a scalar field $\varphi(x)$ on the background of a $(D+1)$-dimensional AdS spacetime with the curvature radius α. The corresponding line element is given by $ds^2_{\rm AdS} = (\alpha/z)^2\,[dt^2 - (dx^1)^2 - d\mathbf{x}^2 - dz^2]$, where $\mathbf{x} = (x^2, \ldots, x^{D-1})$, $x^i \in (-\infty, +\infty)$ and $0 < z < \infty$. The AdS boundary and horizon are presented by the hypersurfaces $z = 0$ and $z = \infty$, respectively. The Ricci scalar and the cosmological constant are expressed by the relations $R = -D(D+1)/\alpha^2$ and $\Lambda = -D(D-1)/(2\alpha^2)$. The operator of the scalar field with the curvature coupling constant ξ obeys the equation $\left(g^{ik}\nabla_i\nabla_k + m^2 + \xi R\right)\varphi(x) = 0$. We are interested in the effects of two branes located at $x^1 = a_1$ and $x^1 = a_2$ on the local properties of vacuum state. It is assumed that on the brane at $x^1 = a_j$, $j = 1, 2$, the field obeys Robin boundary condition $(A_j + B_j n_j^i \nabla_i)\varphi(x) = 0$, where n_j^i is the normal to the brane. In the region between the branes, $a_1 \le x^1 \le a_2$, one has $n_j^i = (-1)^{j-1}\delta_1^i z/\alpha$. The special case with $B_j/A_j = \alpha\beta_j/z$ is considered, where β_j, $j = 1, 2$, are constants. The complete set of mode functions, obeying the boundary conditions on the branes, are found. By using those modes, the positive frequency Wightman function is evaluated. The local VEVs are obtained in the coincidence limit of the arguments of that function and its derivatives.

In the region between the branes, the VEV of the field squared is expressed as

$$\langle\varphi^2\rangle = \langle\varphi^2\rangle_0 + \frac{(\sqrt{\pi}\alpha)^{1-D}}{2^{D+2\nu}}\int_0^\infty dx\, x^{D+2\nu-1} F_\nu^{D/2}(x)\frac{2+\sum_{j=1,2} e^{2|x^1-a_j|x/z} c_j(x/z)}{c_1(x/z)c_2(x/z)e^{2ax/z}-1}, \quad (16)$$

where $\nu = \sqrt{D^2/4 - D(D+1)\xi + m^2\alpha^2}$, $a = a_2 - a_1$, $c_j(\lambda) = (\beta_j\lambda - 1)/(\beta_j\lambda + 1)$, and

$$F_\nu^\mu(u) = \frac{{}_1F_2(\nu+\frac{1}{2};\mu+\nu+\frac{1}{2}, 1+2\nu; -u^2)}{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(1+\nu)}, \quad (17)$$

with ${}_1F_2(a; b, c; z)$ being the hypergeometric function. In (16), $\langle\varphi^2\rangle_0$ is the renormalized VEV in AdS spacetime when the branes are absent. Because of the maximal symmetry of AdS geometry, the part $\langle\varphi^2\rangle_0$ does not depend on the spacetime point.

The VEVs of the nonzero components of the energy-momentum tensor are written in the form (no summation over i)

$$\langle T_i^i\rangle = \langle T_i^i\rangle_0 - \frac{\alpha^{-1-D}}{2^{D+2\nu}\pi^{\frac{D-1}{2}}}\int_0^\infty dx\, x\left\{\frac{E_i x^{D+2\nu}F_\nu^{\frac{D}{2}}(x)}{c_1(x/z)c_2(x/z)e^{2ax/z}-1}\right.$$
$$\left.+\frac{2+\sum_{j=1,2}e^{2|x^1-a_j|x/z}c_j(x/z)}{c_1(x/z)c_2(x/z)e^{2ax/z}-1}\left[A_i x^{D+2\nu}F_\nu^{\frac{D}{2}+1}(x)+\hat{B}_i x^{D+2\nu}F_\nu^{\frac{D}{2}}(x)\right]\right\}, \quad (18)$$

$$\langle T_D^1\rangle = -\frac{2\alpha^{-1-D}}{2^{D+2\nu}\pi^{\frac{D-1}{2}}}\int_0^\infty dx\frac{\sum_{j=1,2}(-1)^j e^{2|x^1-a_j|x/z}c_j(x/z)}{c_1(x/z)c_2(x/z)e^{2ax/z}-1}$$
$$\times\left[\left(\xi-\frac{1}{4}\right)x\partial_x+\xi\right]x^{D+2\nu}F_\nu^{\frac{D}{2}}(x), \quad (19)$$

where $E_i = 2(1-4\xi)$ for $i = 0, 2, \ldots, D$, $E_1 = -2$, $A_1 = 0$, $A_i = 1/2$ for $i = 0, 2, \ldots, D-1$, and $A_D = (1-D)/2$. In the expressions for the diagonal components, $\hat{B}_i$ are the second-order differential operators with respect to x. For example,

$$\hat{B}_1 = (\xi - 1/4)\,\partial_x^2 + [(D-1/4) - (D-2)\,\xi]\,x^{-1}\partial_x - D\xi x^{-2}. \quad (20)$$

In (18), the part $\langle T_i^k\rangle_0$ corresponds to the vacuum energy-momentum tensor in the brane-free AdS spacetime. From the maximal symmetry of the AdS geometry one has $\langle T_i^k\rangle_0 = \mathrm{const}\cdot\delta_i^k$. The components $\langle T_0^0\rangle$ and $\langle T_i^i\rangle$, $i = 2, \ldots, D-1$, determining the energy density and stresses along the directions parallel to the branes (except the component $i = D$), are equal. Of course, that is a consequence of the problem's symmetry. The products $\alpha^{D-1}\langle\varphi^2\rangle$ and $\alpha^{D+1}\langle T_i^k\rangle$ depend on the quantities having dimension of length (x^1, a_j, β_j) and on the coordinate z through the ratios x^1/z, a_j/z, β_j/z. Those ratios are the proper values of the quantities measured by an observer with fixed z in units of the curvature radius α. This feature is a consequence of the AdS maximal symmetry.

In the limit $\alpha \to \infty$, with a fixed value of the coordinate $y = \alpha\ln(z/\alpha)$, from (16) and (18) the corresponding VEVs are obtained in the region between two Robin plates in the background of Minkowski spacetime with the line element $ds_{\mathrm{M}}^2 = dt^2 - (dx^1)^2 - d\mathbf{x}^2 - dz^2$. For a massless field, they are reduced to the results derived in [4]. In the Minkowskian limit, the off-diagonal component $\langle T_D^1\rangle$ tends to zero like $1/\alpha$. Another special case corresponds to a conformally coupled massless field with $\xi = \xi_D \equiv (D-1)/(4D)$. In this case, the problem on the AdS bulk is conformally related to the problem in the Minkowski spacetime with the line element ds_{M}^2, involving two parallel Robin plates at $x^1 = a_1$ and $x^1 = a_2$ intersected by the plate $z = 0$ with the Dirichlet boundary condition. The latter plate is the conformal image of the AdS boundary. The VEVs are connected by simple relations $\langle\varphi^2\rangle = \langle\varphi^2\rangle_0 + (z/\alpha)^{D-1}\langle\varphi^2\rangle_{(\mathrm{M})}$ and $\langle T_k^i\rangle = \langle T_k^i\rangle_0 + (z/\alpha)^{D+1}\langle T_k^i\rangle_{(\mathrm{M})}$, where $\langle\varphi^2\rangle_{(\mathrm{M})}$ and $\langle T_k^i\rangle_{(\mathrm{M})}$ are the VEVs in the Minkowskian problem.

Near the brane at $x^1 = a_j$, $|x^1 - a_j| \ll z$, the VEVs are dominated by the last terms in (16) and (18). Assuming additionally $|x^1 - a_j| \ll |\beta_j|$ (non-Dirichlet boundary conditions), the leading terms in the expansions over the distance from the brane read

$$\langle\varphi^2\rangle \approx \frac{\Gamma\left(\frac{D-1}{2}\right)}{(4\pi)^{\frac{D+1}{2}}}\left(\alpha\left|x^1-a_j\right|/z\right)^{1-D},\ \langle T_0^0\rangle \approx \frac{(1-D)\langle T_1^1\rangle}{(|x^1-a_j|/z)^2} \approx \frac{2D(\xi_D-\xi)\,\Gamma\left(\frac{D+1}{2}\right)}{\pi^{\frac{D+1}{2}}(2\alpha|x^1-a_j|/z)^{D+1}}. \quad (21)$$

For the off-diagonal component one gets $\langle T_D^1\rangle \approx (x^1 - a_j)\langle T_0^0\rangle/z$. For the Dirichlet boundary condition, the corresponding asymptotics differ from (21) by the sign of the right-hand sides. The leading terms (21) coincide with those for plates in Minkowski bulk, with the distance from the plate replaced

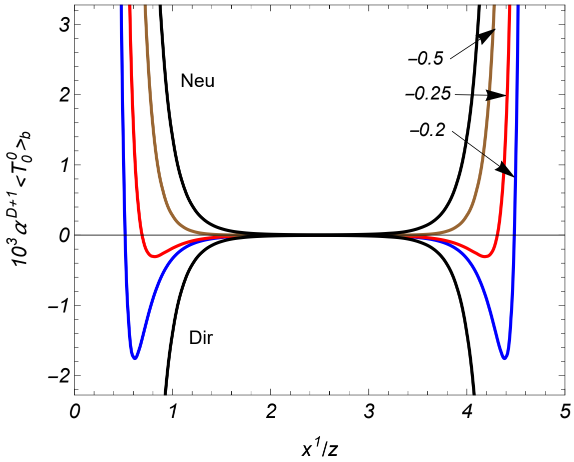

Figure 4: The vacuum energy densities for $D = 4$ minimally coupled scalar fields induced by the branes in the region $0 < x^1/z < 5$.

by the proper distance in AdS bulk. In the region under consideration, the dominant contribution to the VEVs comes from the vacuum fluctuations with small wavelengths, and the influence of the gravitational field on those modes is weak. For points near the AdS boundary and not too close to the branes, corresponding to $z \ll |x^1 - a_j|$, $j = 1, 2$, the brane-induced parts $\langle\varphi^2\rangle - \langle\varphi^2\rangle_0$ and $\langle T_i^i\rangle - \langle T_i^i\rangle_0$ tend to zero like $z^{D+2\nu}$. The off-diagonal component behaves as $\langle T_D^1\rangle \propto z^{D+2\nu+1}$.

Figure 4 presents the brane-induced energy density for minimally coupled scalar fields in the region between the branes versus the proper distance from the brane (in units of α). The graphs are plotted for $a_1 = 0$, $a_2/z = 5$, $m\alpha = 0.5$ and for the same Robin boundary conditions on the branes ($\beta_1 = \beta_2$). The numbers near the graphs correspond to the values of the ratio β_1/z. We have also plotted the graphs for Dirichlet and Neumann boundary conditions ($\beta_j = 0$ and $\beta_j = \infty$, respectively). The vacuum energy density near the branes is positive for a minimally coupled field and non-Dirichlet boundary conditions. For the Dirichlet boundary condition, the energy density is negative. The behaviour of the energy density near the centre with respect to the branes depends on the Robin coefficients. For $\beta_j/z < 0$ and sufficiently close to zero, the brane-induced energy density is negative near the centre. With the increasing value of $|\beta_j|/z$, started from a certain critical value $\beta_j^{(c)}$, that depends on a/z, it becomes positive everywhere in the region between the branes. For the values of the parameters corresponding to Figure 4, the critical value is given by $\beta_j^{(c)}/z \approx -0.69$. The critical value is an increasing function of a/z.

The Casimir forces acting on the branes have two components. The first one corresponds to the normal force, which is determined by the component $\langle T_1^1\rangle$ of the energy-momentum tensor. The vacuum pressure on the brane at $x^1 = a_j$ is given by the expression

$$P_j = \frac{\alpha^{-1-D}}{2^{D+2\nu}\pi^{\frac{D-1}{2}}} \int_0^\infty dx\, x \frac{-2 + [2+c_j(x/z) + 1/c_j(x/z)]\, \hat{B}_1}{c_1(x/z)c_2(x/z)e^{2ax/z} - 1} x^{D+2\nu} F_\nu^{\frac{D}{2}}(x). \tag{22}$$

The normal force is attractive for $P_j < 0$ and repulsive for $P_j > 0$. Unlike the problem in the Minkowskian bulk, the forces for Dirichlet and Neumann boundary conditions ($c_j(u) = -1$ and $c_j(u) = 1$, respectively) are different. Another difference is that the forces acting on separate branes differ if the coefficients in the Robin boundary conditions on them are different. The normal forces can be attractive or repulsive depending on the boundary conditions and the separation between the branes. The effects of background curvature are weak at small separations, and the force is well approximated by the corresponding result for the Minkowski bulk. They are repulsive for the Dirichlet boundary condition on one brane and non-Dirichlet condition on the other and attractive in the remaining cases. The influence of gravity is essential for proper separations larger than the AdS curvature

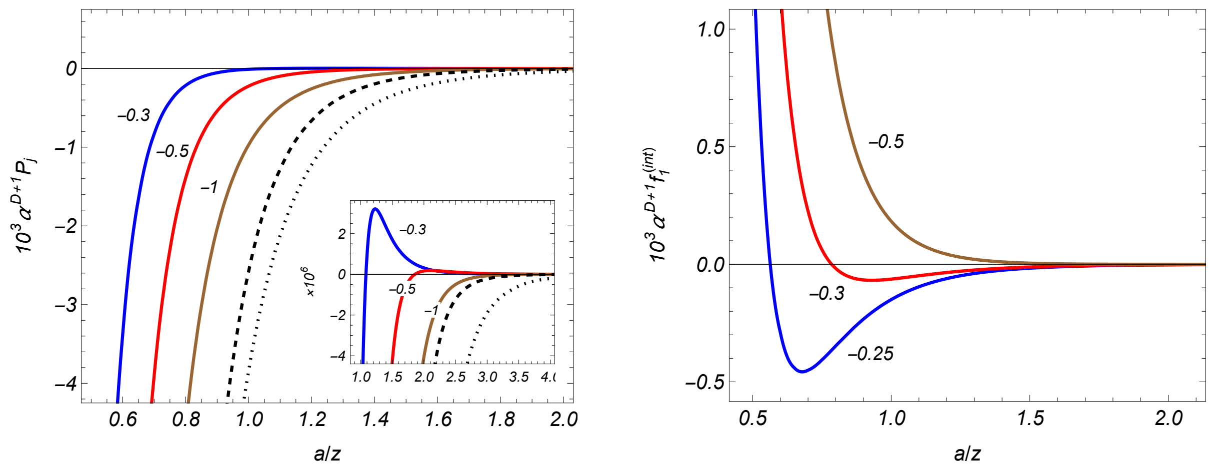

Figure 5: The normal and shear forces per unit surface of the brane $x^1 = a_1$ versus the interbrane separation.

radius, $a > z$. For the brane with Neumann boundary condition, the Casimir force on that brane decays at large separations like $(z/a)^{D+2\nu}$ regardless of the boundary condition on the second brane. For non-Neumann boundary conditions on the brane at $x^1 = a_j$ and for $a \gg |\beta_j|$ the corresponding force behaves as $(z/a)^{D+2\nu+2}$ and the suppression is stronger. The decay of the forces at large separations obeys a power-law for both cases of massless and massive fields. For massive fields, these results contrast the exponential decay in the Minkowski bulk. The normal Casimir forces can be attractive or repulsive at large distances depending on the boundary conditions. The sign of the forces is determined by the factor $4\nu B_\nu \beta_j^2/z^2 + 1$, where $B_\nu = (D + 2\nu + 1)\,\xi - D/4 - \nu/2$. This factor is positive near the horizon and is negative near the AdS boundary if $B_\nu < 0$. This shows that the vacuum pressure changes the sign as a function of z.

A qualitatively different feature of the AdS bulk problem is the vacuum shear force on the branes acting along the z-direction. It is determined by the off-diagonal component $\langle T_1^D \rangle$, evaluated at the location of the brane, and is decomposed into self-action and interaction contributions. The shear force per unit surface of the brane at $z = z_j$ induced by the second brane (the interaction contribution), is expressed as

$$f_j^{(\mathrm{int})} = -\frac{2\alpha^{-1-D}}{2^{D+2\nu}\pi^{\frac{D-1}{2}}} \int_0^\infty dx \frac{c_j(x/z) - 1/c_j(x/z)}{c_1(x/z)c_2(x/z)e^{2ax/z} - 1} \left[\left(\xi - \frac{1}{4} \right) x\partial_x + \xi \right] x^{D+2\nu} F_\nu^{\frac{D}{2}}(x). \quad (23)$$

This part acting on the brane at $x^1 = a_j$ vanishes for Dirichlet and Neumann boundary conditions on that brane regardless of boundary conditions on the second brane. The shear force is directed toward the horizon for $f_j^{(\mathrm{int})} > 0$ and toward the AdS boundary for $f_j^{(\mathrm{int})} < 0$. At small proper separations compared with the curvature radius, $a/z \ll 1$, and for $\xi \neq \xi_D$ one has $f_j^{(\mathrm{int})} \propto (z/a)^D$. At small separations, the shear component of the force has opposite signs for Dirichlet and non-Dirichlet boundary conditions on the second brane. At large proper separations, $a/z \gg 1$, the interaction force behaves as $f_j^{(\mathrm{int})} \propto (z/a)^{D+2\nu+1}$. It has opposite signs for Neumann and non-Neumann boundary conditions on the second brane. For conformally and minimally coupled fields and for $\beta_j < 0$, at large separations between the branes the shear force acting on the brane $x^1 = a_j$ is directed toward the AdS horizon for Neumann boundary condition on the second brane and toward the AdS boundary for non-Neumann conditions.

In figure 5, the normal and shear forces are presented versus the proper separation between the branes, in units of the AdS curvature radius, for $D = 4$ minimally coupled scalar field. The same boundary conditions are imposed on the branes, and the numbers near the curves are the values for $\beta_1/z = \beta_2/z$. The graphs are plotted for $m\alpha = 0.5$. The dashed and dotted curves on the left panel correspond to Dirichlet's and Neumann's boundary conditions, respectively. The graphs on that panel show that the forces attractive at small separations may become repulsive for larger distances. The shear force is directed toward the horizon at small separations between the branes and toward the AdS boundary at large separations.

CONCLUSIONS

1. Expressions are derived for the Hadamard function and currents in the Fulling-Rindler for a charged scalar field in Rindler spacetime with a part of spatial dimensions compactified to a torus. The charge and current densities along uncompact dimensions vanish. The current density along compact dimensions is a periodic function of the magnetic flux enclosed by those dimensions and vanishes on the Rindler horizon. For small accelerations of the Rindler observer, the difference in the current densities for the Fulling-Rindler and Minkowski vacua is exponentially small.

2. Properties of the fermionic Fulling-Rindler vacuum for a massive Dirac field are investigated in a general number of spatial dimensions. The fermion condensate vanishes for a massless field and is negative for non-zero mass. Unlike the case of scalar fields, the fermionic vacuum stresses are isotropic for the general case of massive fields. The vacuum energy density and the pressures are negative. The corresponding spectral distributions exhibit thermal properties with the standard Unruh temperature for a massless field. However, the density-of-states factor is not Planckian for a general number of spatial dimensions. The thermal distribution is of the Bose-Einstein type in an even number of spatial dimensions. In an even number of space dimensions, the fermion condensate and the mean energy-momentum tensor coincide for the fields, realizing two inequivalent irreducible representations of the Clifford algebra.

3. Effects of a generalized cosmic string type defect on the vacuum expectation value of the energy-momentum tensor are investigated for the electromagnetic field in locally dS spacetime for a general number of spatial dimensions D. The topological contributions are explicitly extracted in the diagonal and off-diagonal components for the Bunch-Davies vacuum state. The latter describes the presence of radially directed energy flux in the vacuum state. It vanishes for $D = 3$ and is directed towards the cosmic string for $D \geq 4$. The topological contributions in the vacuum stresses are anisotropic and, unlike the geometry of a cosmic string in the Minkowski spacetime, for $D > 3$, the stresses along the directions parallel to the string core differ from the energy density. The corresponding expectation values can be positive or negative depending on the planar angle deficit and the distance from the cosmic string.

4. Near the cosmic string, the effect of the gravitational field on the diagonal components of the topological part is weak, and the leading terms in the expansions of the diagonal components for the energy-momentum tensor coincide with the expectation values for a cosmic string in the background of Minkowski spacetime. The spacetime curvature modifies the topological terms' behaviour at proper distances from the cosmic string larger than the dS curvature radius. In that region, the topological contributions in the diagonal components of the energy-momentum tensor decay in inverse proportion to the fourth power of the proper distance, and the energy flux density behaves as the inverse-fifth power for all values of the spatial dimension. The exception is the energy density in the special case $D = 4$. The energy flux is absent for a cosmic string in

the Minkowski bulk, and the diagonal components are proportional to the $(D+1)$th power of the inverse distance.

5. For a massive scalar field with general curvature coupling, the Wightman function is evaluated in the geometry of two parallel branes perpendicular to the AdS boundary. The vacuum energy-momentum tensor, in addition to the diagonal components, has a non-zero off-diagonal stress. Depending on the boundary conditions and also on the distance from the branes, the vacuum energy density can be either positive or negative. The Casimir forces acting on the branes have two components. The first corresponds to the standard normal force, and the second is parallel to the branes and presents the vacuum shear force.

6. Unlike the problem of parallel plates in the Minkowski bulk, the normal Casimir forces acting on separate branes differ if the boundary conditions on the branes are different. They can be either repulsive or attractive. Similarly, depending on the coefficients in the boundary conditions, the shear force is directed toward or from the AdS boundary. The separate components may also change their signs as functions of the interbrane separation. At large proper separations between the branes, compared to the AdS curvature radius, both of the components of the Casimir forces exhibit a power-law decay. For a massive scalar field, this behaviour is in contrast to that for the Minkowski bulk, where the decrease is exponential.

REFERENCES

1. E. R. Bezerra de Mello, A. A. Saharian, Finite temperature current densities and Bose-Einstein condensation in topologically nontrivial spaces. Phys. Rev. D **87**, 045015 (2013).

2. E. R. Bezerra de Mello, A. A. Saharian, V. Vardanyan, Induced vacuum currents in anti-de Sitter space with toral dimensions. Phys. Lett. B **741**, 155-162 (2015).

3. P. Candelas, D. Deutsch, Fermion fields in accelerated states. Proc. Roy. Soc. London A **362**, 251-262 (1978).

4. A. Romeo, A. A. Saharian, Casimir effect for scalar fields under Robin boundary conditions on plates. J. Phys. A **35**, 1297-1320 (2002).

PUBLICATIONS ON THE TOPIC OF THE THESIS

1. S. Bellucci, A. A. Saharian, V. Kh. Kotanjyan, Vacuum densities and the Casimir forces for branes orthogonal to the AdS boundary. Phys. Rev. D **106**, 065021 (2022).

2. V. Kh Kotanjyan, A. A. Saharian, M. R. Setare, Vacuum currents in partially compactified Rindler spacetime with an application to cylindrical black holes. Nucl. Phys. B **980**, 115838 (2022).

3. A. A. Saharian, et al. Electromagnetic vacuum stresses and energy fluxes induced by a cosmic string in de Sitter spacetime. International Journal of Modern Physics D 32.07 (2023): 2350042.

4. V. Kh. Kotanjyan, Polarization of the Fulling-Rindler vacuum in models with compact dimensions. Arm. J. Phys. **16**, 83-90 (2023).

5. S. Bellucci, V. Kh. Kotanjyan, A. A. Saharian, Fermionic condensate and the mean energy-momentum tensor in the Fulling-Rindler vacuum. Phys. Rev. D **108**, 085014 (2023).

ԱՄՓՈՓԱԳԻՐ

Ատենախոսությունում հետազոտված են քվանտային վակուումի հատկությունները ոչ իներցիալ համակարգերում, ոչ-տրիվիալ տոպոլոգիայով դե Սիտտերի տարածությունում ու անտի-դե Սիտտերի տարածությունում, երբ առկա են նաև սահմաններ: Որպես վակուումի լոկալ բնութագրեր, դիտարկված են դաշտի քառակուսու, հոսանքի խտության և էներգիա-իմպուլսի թենզորի միջինները: Վակուումային միջիններում բացահայտ կերպով առանձնացված են տոպոլոգիական ու սահմաններով մակածված մասերը և հետազոտված է դրանց վարքը պարամետրերի արժեքների սահմանային տիրույթներում: Քննարկված են կիրառությունները տիեզերագիտությունում, Կալուցա-Քլայնի և բրան աշխարհների մոդելներում:

1. Արտածվել են Ֆուլինգ-Ռինդլերի վակուումային վիճակում գտնվող լիցքավորված սկալյար դաշտի Հադամարի ֆունկցիայի և հոսանքի խտության արտահայտությունները տորոիդալ կոմպակտ ենթատարածությամբ Ռինդլերի տարածաժամանակում: Լիցքի խտությունը և հոսանքի բաղադրիչները ոչ-կոմպակտ չափերի երկայնքով զրոյանում են: Կոմպակտ չափի երկայնքով հոսանքի խտութտունը պարբերական ֆունկցիա է դրանով պարփակված մագնիսական հոսքից և զրոյանում է Ռինդլերի հորիզոնի վրա: Փոքր արագացումների դեպքում Ֆուլինգ-Ռինդլերի և Մինկովսկու վակուումներում հոսանքի խտությունների տարբերությունը էքսպոնենցիալ մարում է:

2. Զանգվածեղ Դիրակյան դաշտի համար հետազոտվել են ֆերմիոնային Ֆուլինգ-Ռինդլերի վակուումի հատկությունները տարածության չափողականության ընդհանուր դեպքում: Ֆերմիոնային կոնդենսատը զրոյանում է անզանգված դաշտի համար և բացասական է զանգվածեղ դաշտի դեպքում: Ի տարբերություն սկալյար դաշտի դեպքի, ֆերմիոնային վակուումի լարվածությունները իզոտրոպ են զանգվածեղ դաշտի ընդհանուր դեպքի համար: Վակուումային էներգիայի խտությունը և ճնշումները բացասական են: Զրոյական զանգվածով դաշտի դեպքում համապատասխան սպեկտրալ բաշխումները ջերմային բնույթի են՝ ստանդարտ Ունրուի ջերմաստիճանով: Սակայն, վիճակների խտությանը համապատասխանող գործակիցը տարածական չափողականության ընդհանուր դեպքում Պլանկյան չէ: Ջերմային բաշխումը Բոզե-Այնշթայնի տիպի է զույգ թվով տարածական չափողականություններում: Զույգ թվով տարածական չափողականություններում ֆերմիոնային կոնդենսատը և էներգիա-իմպուլսի թենզորի միջինը համընկնում են Քլիֆորդի հանրահաշվի ոչ համարժեք չբերվող ներակայցումների իրացնող դաշտերի համար:

3. Հետազոտվել է $D + 1$ չափանի լոկալ դե Սիտտերի տարածաժամանակում ընդհանրացված կոսմիկական լարի տիպի արատի ազդեցությունը էլեկտրամագնիսական դաշտի էներգիա-իմպուլսի թենզորի վակուումային միջինի վրա: Բանչ-Դեվիսի վակուումային վիճակի համար բացահայտ տեսքով անջատված են տոպոլոգիական ներդրումները ինչպես անկյունագծային, այնպես էլ ոչ-անկյունագծային բաղադրիչներում: Վերջինս նկարագրում է վակուումային վիճակում էներգիայի հոսք շառավղային ուղղությամբ: Այն զրոյանում է $D = 3$ դեպքում, և ուղղված է դեպի կոսմիկական լարը $D \geq 4$ դեպքերում: Տոպոլոգիական ներդրումները վակուումային լարվածություններում անիզոտրոպ են և, ի տարբերություն Մինկովսկու տարածաժամանակում կոսմիկական լարի երկրաչափության, երբ $D > 3$ կոսմիկական լարի առանցքին զուգահեռ ուղղություններով

լարվածությունները տարբեր են էներգիայի խտությունից: Կախված հարթ անկյան դեֆիցիտից և լարից ունեցած հեռավորությունից համապատասխան միջինները կարող են լինել ինչպես դրական, այնպես էլ բացասական:

4. Կոսմիկական լարին մոտ կետերում գրավիտացիոն դաշտի ազդեցությունը էներգիա-իմպուլսի թենզորի անկյունագծային բաղադրիչներում տոպոլոգիայով պայմանավորված ներդրումների վրա թույլ է և համապատասխան վերլուծության գլխավոր անդամները համընկնում են Մինկովսկու տարածաժամանակում կոսմիկական լարի համար վակուումային միջինների հետ: Տարածաժամանակի կորությունը էապես փոխում է տոպոլոգիական ներդրումների վարքը կոսմիկական լարից դե Սիտտերի կորության շառավղից մեծ սեփական հեռավորությունների վրա: Այդ տիրույթում էներգիա-իմպուլսի թենզորի անկյունագծային բաղադրիչներում տոպոլոգիական ներդրումները տարածական բոլոր չափողականություններում նվազում են սեփական հեռավորության չորրորդ աստիճանին հակադարձ համեմատական, իսկ էներգիայի հոսքը՝ հինգերորդ աստիճանին հակադարձ համեմատական: Բացառություն է կազմում էներգիայի խտությունը $D = 4$ դեպքում: Մինկովսկու տարածաժամանակում կոսմիկական լարի համար էներգիայի հոսքը բացակայում է, իսկ անկյունագծային բաղադրիչները հակադարձ համեմատական են հեռավորության $(D+1)$-րդ աստիճանին:

5. Ստացվել է Վայթմանի ֆունկցիայի արտահայտությունը կորության հետ ընդհանուր կապի պարամետրով զանգվածեղ սկալյար դաշտի համար անտի-դե Սիտտերի սահմանին ուղղահայաց երկու զուգահեռ բրանների երկրաչափությունում: Վակուումային էներգիա-իմպուլսի թենզորը բացի անկյունագծային բաղադրիչներից ունի նաև զրոյից տարբեր ոչ անկյունագծային բաղադրիչ: Կախված եզրային պայմաններից և բրանների միջև հեռավորությունից՝ վակուումային էներգիայի խտությունը կարող է լինել ինչպես դրական, այնպես էլ բացասական: Բրանների վրա ազդող Կազիմիրի ուժերը ունեն երկու բաղադրիչներ: Առաջինը համապատասխանում է ստանդարտ նորմալ ուժին, իսկ երկրորդը զուգահեռ է բրաններին և ներկայացնում է վակուումային շոշափող ուժ:

6. Ի տարբերություն Մինկովսկու տարածաժամանակում զուգահեռ թիթեղների խնդրի, առանձին բրանների վրա ազդող Կազիմիրի ուղղահայաց ուժերը տարբեր են, եթե տարբեր են դրանց վրա դրվող եզրային պայմանները: Դրանք կարող են լինել ինչպես ձգողական, այնպես էլ վանողական: Նմանապես, կախված եզրային պայմաններում գործակիցներից, շոշափող ուժը կարող է ուղղվել ինչպես դեպի անտի-դե Սիտտերի սահմանը, այնպես էլ հորզոնի ուղղությամբ: Կախված միջբրանային հեռավորությունից առանձին բաղադրիչները կարող են փոխել իրենց նշանը: Անտի-դե Սիտտերի կորության շառավղից մեծ միջբրանային հեռավորությունների վրա Կազիմիրի ուժի երկու բաղադրիչներն էլ նվազում են աստիճանային օրենքով: Զանգվածեղ սկալյար դաշտի համար նման վարքը տարբեր է Մինկովսկու տարածաժամանակում դեպքից, որտեղ նվազումը էքսպոնենցիալ օրենքով է:

КОТАНДЖЯН ВАРДАЗАР

ВАКУУМНЫЕ КВАНТОВЫЕ ЭФФЕКТЫ В НЕИНЕРЦИАЛЬНЫХ СИСТЕМАХ И ГРАВИТАЦИОННЫХ ПОЛЯХ

В диссертации исследованы свойства квантового вакуума в неинерциальных системах отсчета, в пространствах де Ситтера (dS) с нетривиальной топологией и в пространстве анти-де Ситтера (AdS) при наличии границ. В качестве локальных характеристик вакуумного состояния рассмотрены вакуумные средние квадрата поля, плотности тока и тензора энергии-импульса. Явно выделены вклады в вакуумных средних, индуцированные топологией и границами и исследовано их поведение в различных предельных областях значений параметров. Обсуждаются приложения в космологии, в моделях Калуцы-Клейна и бран миров.

1. Получены выражения для функции Адамара и токов в вакууме Фуллинга-Риндлера для заряженного скалярного поля в пространстве-времени Риндлера при наличии тороидально компактифицированных пространственных измерений. Показано что плотности заряда и тока вдоль некомпактных измерений равны нулю, а плотность тока вдоль компактных измерений является периодической функцией магнитного потока, пронизывающего эти измерения, и стремится к нулю на горизонте Риндлера. При малых ускорениях разница плотностей тока для вакуумов Фуллинга-Риндлера и Минковского экспоненциально мала.

2. Исследованы свойства фермионного вакуума Фуллинга-Риндлера для массивного поля Дирака при произвольном числе пространственных измерений. Показано что фермионный конденсат равен нулю для безмассового поля и отрицателен при ненулевой массе. В отличие от случая скалярных полей, фермионные вакуумные натяжения изотропны для общего случая массивных полей. Плотность энергии вакуума и давления отрицательны. Для безмассового поля соответствующие спектральные распределения имеют тепловой характер со стандартной температурой Унру. Однако фактор плотности состояний не является планковским для общего случая числа пространственных измерений. Тепловое распределение имеет Бозе-Эйнштейновский тип в четном числе пространственных измерений. При четном количестве пространственных измерений фермионный конденсат и среднее тензора энергии-импульса совпадают для полей, реализующих два неэквивалентных неприводимых представления алгебры Клиффорда.

3. Исследовано влияние дефекта типа обобщенной космической струны на вакуумное среднее тензора энергии-импульса для электромагнитного поля в локальном пространстве-времени dS для общего числа пространственных измерений D. Для вакуумного состояния Банча-Дэвиса явно выделены топологические вклады как в диагональных, так и в недиагональных компонентах. Последнее описывает наличие в вакуумном состоянии радиально направленного потока энергии. Топологические вклады в вакуумные натяжения анизотропны и, в отличие от геометрии космической струны в пространстве-времени Минковского, при $D > 3$ натяжения вдоль направлений, параллельных оси струны, отличаются от плотности энергии. В зависимости от дефицита плоского угла и расстояния от космической струны соответствующие средние значения могут быть как положительными, так и отрицательными.

4. Вблизи космической струны влияние гравитационного поля на диагональные компоненты топологической части слабое. Главные члены в разложениях диагональных компонент

тензора энергии-импульса совпадают со значениями средних для космической струны на фоне пространства-времени Минковского. Кривизна пространства-времени существенно влияет на поведение топологических членов на собственных расстояниях от космической струны, больших, чем радиус кривизны dS. В этой области топологические вклады в диагональные компоненты тензора энергии-импульса затухают обратно пропорционально четвертой степени собственного расстояния, а плотность потока энергии ведет себя как обратная пятая степень для всех значений пространственной размерности. Исключением является плотность энергии в случае $D = 4$. Для космической струны в пространстве Минковского поток энергии отсутствует, а диагональные компоненты обратно пропорциональны $(D + 1)$-й степени расстояния.

5. Вычислена функция Вайтмана для массивного скалярного поля с произвольным значением параметра связи с кривизной в геометрии двух параллельных бран, перпендикулярных границе AdS. Тензор энергии-импульса вакуума помимо диагональных компонент содержит ненулевое недиагональное напряжение. В зависимости от граничных условий, а также от расстояний до бран, плотность энергии вакуума может быть как положительной, так и отрицательной. Силы Казимира, действующие на браны, имеют две компоненты. Первая соответствует стандартной нормальной силе, а вторая параллельна бранам и представляет собой вакуумную силу сдвига.

6. В отличие от задачи о параллельных пластинах в пространстве Минковского, нормальные силы Казимира, действующие на отдельные браны, различны, если граничные условия на бранах различны. Они могут быть как отталкивающими, так и притягивающими. Аналогичным образом, в зависимости от коэффициентов в граничных условиях, параллельная сила направлена к границе AdS или от нее. Отдельные компоненты также могут менять свой знак в зависимости от межбранного растояния. При больших собственных расстояниях между бранами по сравнению с радиусом кривизны AdS обе компоненты сил Казимира затухают по степенному закону. Для массивного скалярного поля такое поведение отличается от поведения для пространства Минковского, где затухание является экспоненциальным.